２０２２年度実施

慶應義塾大学大学院入試問題

経済学研究科（修士課程）

２０２２年７月９日 実施

科目名	Economics (English)	受験番号	Examination number	氏名	Name

注意事項（Please note:）

1．This set of questions contains ten pages (including the cover page).

2．There are six questions from which you should choose two to answer. Each question should be answered on a separate answer sheet. Please write the number of the question you are answering on each answer sheet.

3．If you answer two or more questions on one answer sheet, only the first answer will be treated as a valid answer. Everything after the first answer will not be marked.

4．Answer in English.

5．Although the question sheets will not be collected after the examination, please write your name and examination number (受験番号, jyuken-bango) on the cover page.

Question 1. Answer only one of the following two questions; A and B. If you answer both, all answers for Question 1 become invalid.

A. Answer all questions in A-1 and A-2.

A-1. Consider a duopoly market, with firm 1 and firm 2. If firm 1 produces q_1 units and firm 2 produces q_2 units, the inverse demand function of this market is $1000 - (q_1 + q_2)$.

Assume that the total cost function of firm 1 is $TC_1(q_1) = 100q_1$, and the total cost function of firm 2 is $TC_2(q_2) = 100q_2$.

(a) Consider a Cournot competition such that the firms choose a non-negative real number q_1 and q_2 simultaneously. Find the Cournot-Nash equilibrium quantity combination. Include the derivation process in your answer.

(b) Assume that firm 1 has the production capacity of 200 units so that it must choose a real number q_1 where $0 \leqq q_1 \leqq 200$. Firm 2 does not have any capacity constraint. All these are common knowledge between the firms. Find the Cournot-Nash equilibrium quantity combination. Include the derivation process in your answer.

(c) Assume that firm 2 does not know the technology of firm 1 completely. Firm 2 believes that, with probability 0.5, firm 1 has no capacity constraint and can choose any non-negative real number as its quantity, and, with probability 0.5, firm 1 must choose a real number such that $0 \leqq q_1 \leqq 200$. Firm 2 does not have any capacity constraint. All these are common knowledge between the firms. Thus, the game is now a Bayesian game.

Let the firm 1 with no capacity constraint be the Unconstrained type and the firm 1 with the production capacity be the Bounded type. Write the quantity of the Unconstrained type as q_{1u} and that of the Bounded type as q_{1b}. Find the Baysian Nash equilibrium $(q_{1u}^*, q_{1b}^*, q_2^*)$. Include the derivation process in your answer.

A-2. (a) Write the definition of a (Pareto) efficient allocation.

(b) Write the definition of a pure public good.

(c) Write as rigorously as possible the condition which guarantees that an allocation is efficient when there is a pure public good in the economy, and the derivation of the condition.

(d) Prove that perfectly competitive markets do not guarantee that the allocation is efficient when there is a pure public good in the economy.

(e) Explain the concept of Lindahl equilibrium and why the allocation of a Lindahl equilibrium is efficient. (You can assume quasi-linear utility functions for consumers, if you wish.)

(f) Point out a problem in implementing a Lindahl equilibrium.

B. Suppose that the relationship between the inflation rate π and the unemployment rate u is described by equation (1) (Phillips curve), and that the central bank's loss function $L(\pi, u)$ is represented by equation (2).

$$\pi = E\pi - \frac{1}{\alpha}(u - u^n), \tag{1}$$

$$L(\pi, u) = 0.5u^2 + 0.5\pi^2. \tag{2}$$

$E\pi$ is the expected inflation rate, u^n is the natural rate of unemployment, and α is a parameter. For simplicity, we assume that the central bank can completely control the inflation rate through monetary policy.

(a) Derive a standard short-term aggregate supply curve using a sticky price model or an incomplete information model, and then show that equation (1) can be derived from the short-term aggregate supply curve.

(b) When the central bank commits to a particular level of inflation under a fixed rule, what is the optimal fixed rule for the central bank? Why is that the optimal fixed rule?

(c) When the central bank conducts discretionary monetary policy, find the optimal inflation rate for the central bank. Then, find the inflation rate that prevails when private agents expect the central bank to choose this optimal inflation rate.

(d) By comparing the unemployment rate and the inflation rate in cases (b) and (c) above, discuss which monetary policy is preferable. Intuitively explain why the consequences of optimal monetary policy differ between (b) and (c).

(e) Find a loss function $L(\pi, u)$ for the central bank such that the optimal monetary policy is the same in (b) and (c) above.

Question 2.

Answer the following three questions on capitalism. Base your answer on the methodology of Marxian economics.

(1) Briefly explain the following concepts.
①Commodities
②Constant capital
③Variable capital
④Extra surplus-value
⑤Organic composition of capital

(2) Discuss the relationships between capital accumulation and technological improvement.

(3) Discuss whether or not the reproduction schema is applicable for analyzing the real economy.

Question 3.

Consider a linear regression model: $Y_i = \beta X_i + u_i$, $i = 1, \ldots, n$, where the random vector $[X_i, u_i]'$ follows an independent and identical distribution over i, and their moments are given as $E[X_i^s u_i^r] = \omega_{s,r}$ for real numbers s and r. In particular,

$$E(X_i) = \omega_{1,0} = \mu_X, \qquad E(u_i) = \omega_{0,1} = 0, \qquad E(X_i\, u_i) = \omega_{1,1} = \mu_{Xu} \neq 0,$$

and for $k > 1$,

$$E(X_i^k) = \omega_{k,0} = \mu_X^{(k)}, \qquad E(u_i^k) = \omega_{0,k} = \mu_u^{(k)}.$$

(Note that X_i and u_i are correlated).

(1) Let the ordinary least squares estimator for β be denoted by $\hat{\beta}_{LS}$. Answer the probability limit of $\hat{\beta}_{LS}$ as $n \to \infty$.

(2) For $\hat{\beta}_{LS}$, it holds that

$$\sqrt{n}\left(\hat{\beta}_{LS} - \beta - \boxed{\text{A}}\right) \to_d N\left(\boxed{\text{B}}, \boxed{\text{C}}\right),$$

where "$\to_d$" implies the convergence in distribution. Provide the appropriate numbers or expressions for A, B, and C.

(3) Let Z_i be a random variable such that the random vector $[Z_i, X_i]'$ is independently and identically distributed over i with $E(Z_i) = \mu_Z$, $E(Z_i^2) = \mu_Z^{(2)}$, $E(Z_i X_i) = \mu_{ZX} \neq 0$, and be independent of u_j for all j. Derive the instrumental variables estimator $\hat{\beta}_{IV}$ for β, using Z_i as an instrument.

(4) For $\hat{\beta}_{IV}$ derived in (3), it holds that

$$\sqrt{n}\left(\hat{\beta}_{IV} - \beta - \boxed{\text{D}}\right) \to_d N\left(\boxed{\text{E}}, \boxed{\text{F}}\right)$$

where "$\to_d$" implies the convergence in distribution. Provide the appropriate numbers or expressions for D, E, and F.

Let U_i, $i = 1, \ldots, n$ ($n \geq 1$) be a sample of size n and $U_i \sim U(0, a)$, where $U(0, a)$ denotes a uniform distribution over $(0, a)$, and $a > 0$ is a real number. Consider $\hat{a}_n = \max\{U_1, \ldots, U_n\}$ as an estimator for the unknown parameter a.

(5) Derive the distribution function and probability density function of $\hat{a}_n$.

(6) Derive the variance of $\hat{a}_n$.

(7) Derive the mean squared error of $\hat{a}_n$.

(8) Consider $\hat{a}_n^* \equiv 2\bar{U}$ as a new estimator for a, where $\bar{U}$ is the sample mean of U_i, $i, \ldots, n$.
Choose all of the statements about $\hat{a}_n$ that are correct, out of the following four statements.
(A) It is an unbiased estimator. (B) It is a consistent estimator.
(C) It converges in mean square to a.
(D) The mean squared error of $\hat{a}_n$ is equal to or less than that of $\hat{a}_n^*$.

Question 4.

Answer one of A, B, and C. If you answer two or more, all answers become invalid.

A

Suppose there is a risk-free asset with a certain rate of return 1%. There are also risky stocks a and b that will yield state-contingent returns shown in the table below.

	Future state of the economy		
	State 1 (probability 1/3)	State 2 (probability 1/3)	State 3 (probability 1/3)
Stock a	1%	2%	3%
Stock b	3%	0%	3%

(1) Derive the equation for the efficient frontier of portfolios consisting of only stocks a and b, and draw a graph of the frontier.

(2) Derive the equation for the efficient frontier of portfolios consisting of a, b and the risk-free asset, and draw a graph of the frontier. How would risk-averse investors allocate their wealth between these assets? Explain. (You don't need to compute the exact portfolio weights. Graphical explanations are enough.)

(3) Is the following statement true, false, or neither true nor false for certain? Explain your reasoning. "The central bank's interest rate cut (i.e., decrease in the risk-free rate) will be welcomed by investors because it increases their portfolios' Sharpe ratio."

B

Consider the relationship between location and housing rent in a city. All employment is concentrated in the central business district (CBD), a single point in the space. Each individual living in this city makes a round-trip commute to the CBD, where she works and earns a fixed income of y. We assume that space is featureless, meaning that any location in this city is identical except for the distance to the CBD, which we denote x. An individual has a utility function $u(h,z) = \ln(h) + \ln(z)$, where h is her housing consumption, z is her consumption of other goods, and $\ln(\cdot)$ represents natural logarithm. Rent per unit of housing consumption is r, and the price of other goods is normalized to 1. We assume that commuting involves only a monetary cost, which is proportional to the distance to the CBD. The cost of commuting is then represented by tx, where $t > 0$ is a fixed round-trip commuting cost per unit distance.

(1) Consider an individual located at x. Define her utility maximization problem and compute the optimal housing consumption and the optimal consumption of other goods.

(2) Suppose that all individuals are identical, i.e., they have the same preferences, earn the same income y, and face the same per-distance commuting cost t. In equilibrium, all individuals attain the same utility regardless of their location. The equilibrium utility level is denoted by $\bar{u}$. Derive the equilibrium housing rent as a function of x and show how housing rent changes as one moves away from the CBD, assuming $y - tx > 0$. Also explain the economic intuition of your results.

(3) Suppose instead that there are two types of individual, where type H earns wage y^H and type L earns wage y^L. Assume $y^H > y^L$. If these two types of individual both live in this city in equilibrium, what are their equilibrium location choices and the equilibrium housing rent in this city? Explain.

C

Consider the environmental regulation for an economy consisting of two polluters ($i = A, B$). The polluters emit a uniformly mixed pollutant (i.e., the damages from pollution do not depend on when and where they emit the pollutant). Their emissions levels in the absence of regulation are, respectively, $e_A = 100$, $e_B = 145$. Assume that the polluters are rational cost-minimizers with the following abatement cost functions:

$$C_A(q_A) = 5q_A + \frac{q_A^2}{2}$$

$$C_B(q_B) = 5q_B + \frac{q_B^2}{4}$$

where q_i is each polluter i's emissions abatement level. Assume the social benefit from reducing the pollution is estimated as:

$$B(Q,\eta) = 50Q - \frac{Q^2}{3} + \eta$$

where Q is the total emissions abatement $Q = \sum_i q_i$ and η is an error term, which follows a standard normal distribution $N(0,1)$. Answer the following questions.

(1) Suppose the regulatory authority adopts an emissions tax policy. Solve for the optimal tax level that maximizes the expected social welfare, and each polluter's emissions at that tax level.

(2) Suppose, instead, the regulatory authority adopts an emissions trading policy. The authority allocates emissions permits for free, with a uniform allocation rule.

(a) Solve for the optimal cap on total emissions that would maximize the expected social welfare.

(b) Find the competitive equilibrium price of permits and net sales of permits by each polluter.

(c) Use a diagram to illustrate the economic gain from trading of permits.

(3) In the model above, the two policies in (1) and (2) achieve the same expected social welfare. Economists identified several important conditions under which such an equivalence may not hold. Discuss at least two of such conditions and their policy implications.

(4) Assume the authority implements an offset credit system on top of the policy in part (2). Explain what the offset credit system means in this context, and discuss briefly the pros and cons of the system in encouraging polluters' abatement behavior.

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Question 5.

Choose any region or country and discuss the role played by its textile industry in its economic development. Make sure to use concrete historical facts and discuss from the perspective of economic history.

Question 6. Choose one of the themes below and outline its history (if you choose two or more, all answers will be invalid):

(1) Theory of the perfectly competitive market
(2) Welfare economics and social choice theory
(3) Macroeconomics
(4) Theory of international trade
(5) English classical economics
(6) Theory of social contract
(7) Utilitarianism